## Contents

Pr	Preface to the Second Edition		
Pr	eface	to the First Edition	ix
1	Intr	oduction	1
	1.1	A Simple System Generating a Density of States	1
	1.2	The Evolution of Densities: An Intuitive Point of View	5
	1.3	Trajectories Versus Densities	9
		Exercises	13
2	The	Toolbox	17
~	2.1	Measures and Measure Spaces	17
	2.2	Lebesgue Integration	19
	2.3	Convergence of Sequences of Functions	31
	2.0	Exercises	35
3	Mar	kov and Frobenius–Perron Operators	37
	3.1	Markov Operators	37
	3.2		41
	3.3	The Koopman Operator	47
		Exercises	49
4	Studying Chaos with Densities		51
	4.1	Invariant Measures and Measure-Preserving	
		Transformations	51

••	~	
XII	Contents	
All	COHICHIO	

	4.2	Ergodic Transformations	59
	4.3	Mixing and Exactness	65
	4.4	Using the Frobenius-Perron Koopman Operators for	
		Classifying Transformations	71
	4.5	Kolmogorov Automorphisms	79
		Exercises	83
5	The	Asymptotic Properties of Densities	85
	5.1	Weak and Strong Precompactness	86
	<b>5.2</b>	Properties of the Averages $A_n f$	88
	5.3	Asymptotic Periodicity of $\{P^n f\}$	95
	5.4	The Existence of Stationary Densities	100
	5.5	Ergodicity, Mixing, and Exactness	102
	5.6	Asymptotic Stability of $\{P^n\}$	105
	5.7	Markov Operators Defined by a Stochastic Kernel	112
	5.8	Conditions for the Existence of Lower-Bound Functions	123
	5.9	Sweeping	125
	5.10	The Foguel Alterative and Sweeping	129
		Exercises	136
6	The	Behavior of Transformations on Intervals	
	and	Manifolds	139
	6.1	Functions of Bounded Variation	139
	6.2	Piecewise Monotonic Mappings	144
	6.3	Piecewise Convex Transformations with a Strong Repellor	153
	6.4	Asymptotically Periodic Transformations	156
	6.5	Change of Variables	165
	6.6	Transformations on the Real Line	172
	6.7	Manifolds	175
	6.8	Expanding Mappings on Manifolds	183
		Exercises	187
7	Cont	inuous Time Systems: An Introduction	189
	7.1	Two Examples of Continuous Time Systems	190
	7.2	Dynamical and Semidynamical Systems	191
	7.3	Invariance, Ergodicity, Mixing, and Exactness in	
		Semidynamical Systems	195
	7.4	Semigroups of the Frobenius-Perron and Koopman	
		Operators	199
	7.5	Infinitesimal Operators	205
	7.6	Infinitesimal Operators for Semigroups Generated by	
		Systems of Ordinary Differential Equations	210
	7.7	Applications of the Semigroups of the Frobenius-Perron	
		and Koopman Operators	215
	7.8	The Hille-Yosida Theorem and Its Consequences	226

		Contents	xiii		
	7.9	Further Applications of the Hille-Yosida Theorem	232		
	7.10	The Relation Between the Frobenius-Perron and			
		Koopman Operators	241		
	7.11	Sweeping for Stochastic Semigroups	244		
	7.12	Foguel Alternative for Continuous Time Systems	246		
		Exercises	247		
8	Discrete Time Processes Embedded in Continuous				
	Time	Systems	<b>251</b>		
	8.1	The Relation Between Discrete and Continuous Time			
		Processes	251		
	8.2	Probability Theory and Poisson Processes	<b>252</b>		
	8.3 8.4	Discrete Time Systems Governed by Poisson Processes The Linear Boltzmann Equation: An Intuitive	258		
		Point of View	261		
	8.5	Elementary Properties of the Solutions of the Linear			
		Boltzmann Equation	264		
	8.6	Further Properties of the Linear Boltzmann Equation	<b>268</b>		
	8.7	Effect of the Properties of the Markov Operator on			
		Solutions of the Linear Boltzmann Equation	270		
	8.8	Linear Boltzmann Equation with a Stochastic Kernel	273		
	8.9	The Linear Tjon-Wu Equation	277		
		Exercises	280		
9	Entre	рру	283		
	9.1	Basic Definitions	283		
	9.2	Entropy of $P^n f$ When $P$ is a Markov Operator	289		
	9.3	Entropy $H(P^n f)$ When P is a Frobenius-Perron			
		Operator	<b>292</b>		
	9.4	Behavior of $P^n f$ from $H(P^n f)$	395		
		Exercises	300		
10	Stock	nastic Perturbation of Discrete Time Systems	303		
	10.1	Independent Random Variables	304		
	10.2		306		
	10.3	Stochastic Convergence	311		
	10.4	Discrete Time Systems with Randomly Applied Stochastic Perturbations	315		
	10.5	Discrete Time Systems with Constantly Applied Stochastic Perturbations	320		
	10.6	Small Continuous Stochastic Perturbations of Discrete Time Systems	327		
	10.7	Discrete Time Systems with Multiplicative Perturbations Exercises	330 333		

## xiv Contents

11	Stock	nastic Perturbation of Continuous Time Systems	335
	11.1	One-Dimensional Wiener Processes (Brownian Motion)	335
	11.2	d-Dimensional Wiener Processes (Brownian Motion)	344
	11.3	The Stochastic Itô Integral: Development	346
	11.4	The Stochastic Itô Integral: Special Cases	351
	11.5	Stochastic Differential Equations	355
	11.6	The Fokker-Planck (Kolmogorov Forward) Equation	359
	11.7	Properties of the Solutions of the Fokker-Planck	
		Equation	364
	11.8	Semigroups of Markov Operators Generated by Parabolic	
		Equations	<b>36</b> 8
	11.9	Asymptotic Stability of Solutions of the Fokker-Planck	
		Equation	371
	11.10	An Extension of the Liapunov Function Method	378
	11.11	Sweeping for Solutions of the Fokker-Planck Equation	386
	11.12	Foguel Alternative for the Fokker-Planck Equation	388
		Exercises	391
		appent, pelli est est terre en especialiste en la contra la la serie de la contra la contra la contra la contra	
<b>12</b>		cov and Foias Operators	393
	12.1	The Riesz Representation Theorem	393
	12.2	Weak and Strong Convergence of Measures	397
	12.3	Markov Operators	405
	12.4	Foias Operators	411
	12.5	Stationary Measures: Krylov–Bogolubov Theorem for	
		Stochastic Dynamical Systems	417
	12.6	Weak Asymptotic Stability	420
	12.7	Strong Asymptotic Stability	425
	12.8	Iterated Function Systems and Fractals	432
		Exercises	447
Re	ferenc	ees	449
No	tation	and Symbols	457
Inc	lex		461