
Contents

1	Introduction	1
1.1	The Theory of the Circle	1
1.2	Overview of the Book	7
1.3	Preliminaries, and Some Notation	12
Part I Algebraic Curves		
2	Algebra	17
2.1	Polynomials and Power Series	17
2.2	Unique Factorisation	25
2.3	Groups	35
2.4	Linear Algebra Over Integral Domains	41
2.5	Further Exercises	48
3	Affine Space	55
3.1	Definition of Hypersurfaces	56
3.2	The Resultant	58
3.3	Study's Lemma	64
3.4	Affine Lines and Rational Parameterisations	66
3.5	Further Exercises	69
4	Projective Space	73
4.1	Homogeneous Polynomials	74
4.2	Projective Space	76
4.3	Projective Lines and Maps	78
4.4	Embedding Affine Space into Projective Space	81
4.5	Changes of Coordinates	87
4.6	Spaces of Curves	92
4.7	Products of Projective Spaces	95
4.8	Further Exercises	99
5	Tangents	105
5.1	Introduction: Affine Tangents and Intersections with Lines	105
5.2	Formal Partial Derivatives	110
5.3	Higher Order Tangents	113

5.4	The Intersection of a Line with a Curve	120
5.5	Further Exercises	127
6	Bézout's Theorem	133
6.1	A First Look at the Intersection of Curves	134
6.2	The Homogeneous Resultant	139
6.3	Multiplicity of Intersection and Bézout's Theorem	143
6.4	Coincidence with Earlier Definitions	148
6.5	Categoricity of Multiplicity of Intersection	151
6.6	Affine Calculations	158
6.7	Multiplicities, Orders and Tangents	159
6.8	Further Exercises	161
7	The Elliptic Group	167
7.1	Flexes	168
7.2	The Group Operation on a Nonsingular Cubic Curve	171
7.3	Normal Forms for Nonsingular Cubics	178
7.4	Further Exercises	183
Part II Riemann Surfaces		
8	Quasi-Euclidean Spaces	191
8.1	Topology of \mathbb{R}^n	192
8.2	Manifolds	194
8.3	Compactness	204
8.4	Quotients by Discrete Subgroups	212
8.5	Further Exercises	217
9	Connectedness, Smooth and Simple	221
9.1	Connectedness, Path and Simple	222
9.2	Lifting Maps	226
9.3	Differentiability: A Reminder	230
9.4	Differentiable Manifolds	239
9.5	Partitions of Unity	241
9.6	Differentiable Connectedness	245
9.7	Further Exercises	249
10	Path Integrals	255
10.1	Integrating Forms Along Paths	255
10.2	Integrating Along Smooth Paths	260
10.3	Integrating Vector Fields	266
10.4	Symmetric Vector Fields	270
10.5	Further Exercises	276
11	Complex Differentiation	281
11.1	Complex Derivatives and Integrals	281
11.2	Cauchy's Integral Formula	286
11.3	Uniform Convergence and Power Series	291

11.4	Analytic Functions	296
11.5	Morera, Weierstrass, Liouville	303
11.6	Further Exercises	305
12	Riemann Surfaces	311
12.1	Holomorphic Surfaces	312
12.2	The Open Mapping Theorem	317
12.3	Compact Riemann Surfaces	324
12.4	Riemann Surfaces for the Logarithm and Roots	326
12.5	Analytic Continuation	330
12.6	Differential Forms on Surfaces	332
12.7	Further Exercises	339
Part III Curves and Surfaces		
13	Curves Are Surfaces.....	347
13.1	The Implicit Function Theorem	347
13.2	Nonsingular Curves Are Riemann Surfaces	350
13.3	Intersections with Lines, Revisited	358
13.4	Further Exercises	366
14	Elliptic Functions and the Isomorphism Theorem.....	371
14.1	Elliptic Functions	371
14.2	The Curve E_Γ and the Isomorphism Theorem	379
14.3	Inversion	383
14.4	Further Exercises	389
15	Puiseux Theory	395
15.1	Fractional Power Series and Their Holomorphic Functions	396
15.2	Parameterisations of a Curve	402
15.3	Branches and Places	407
15.4	Puiseux Expansions and Factorisation into Places	412
15.5	Intersection Multiplicities Using Places	416
15.6	Further Exercises	426
16	A Brief History of Elliptic Functions	431
Bibliography		439
Index		441