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Zero-Sound Condensate in a Fermi Liquid[1, 2]

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Abstract—We consider a normal Fermi liquid with a local scalar interaction given by the Landau parameter f_0. The system becomes unstable for $f_0 < -1$ against a growth of scalar-mode excitations (Pomeranchuk instability). We show that the instability may be tamed by the formation of a static Bose condensate of the scalar modes. We study a possible reconstruction of the isospin-symmetric nuclear matter owing to the appearance of the condensate. Possibility of a novel metastable state at subnuclear densities is demonstrated.

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1. INTRODUCTION

Consider a normal Fermi liquid of non-relativistic fermions stable against pairing [1]. The particle–hole (p–h) scattering amplitude on the Fermi surface is determined by the infinite series of p–h diagrams dressing the local interaction separated in scalar and spin channels as $\hat{\Gamma}^{\omega} = \Gamma_0^{\omega}\sigma_0'\sigma_0 + \Gamma_1^{\omega}(\vec{\sigma}'\vec{\sigma})$ with the unit, σ_0, and Pauli's, $\vec{\sigma}$, matrices. On the Fermi surface $\Gamma_{0,1}$ are functions of the angle θ between momenta of incoming and outgoing fermions. $\Gamma_{0,1}^{\omega}$ are expanded in Legendre polinomials, $a^2N\Gamma_0^{\omega}(\theta) = f(\theta) = \sum_l (2l+1) f_l P_l(\cos\theta)$. Here $a \le 1$ determines a quasiparticle weight in the fermion Green's function, $N = \nu m^* p_{\rm F}/\pi^2$ is the density of states at the Fermi surface, m^* is the effective fermion mass. The Fermi momentum $p_{\rm F}$ is related to the total fermion density as $n = \nu p_{\rm F}^3/(3\pi^2)$ with $\nu = 1$ for one type of fermions and $\nu = 2$ for two types of fermions, like for the isospin-symmetric nuclear matter. The Landau parameters f_l can be calculated or fitted from experiments. The scalar parameter f_0 is related to the incompressibility of the system $K_{\rm f} = n d^2E_{\rm f}/dn^2 = (1+f_0)\,p_{\rm F}^2/3m_{\rm F}^*$, where $E_{\rm f}$ is the energy density of the fermion system. The p–h scattering amplitude induced by the f_0 parameter is [1] $T_{\rm ph,0}(\omega,k) = [a^2N(f_0^{-1} + \Phi(\omega,k))]^{-1}$, $\Phi(\omega,k) = \frac{1}{2} + \sum_{i=\pm}(-i)\frac{z_i^2-1}{4(z_+-z_-)}\ln\frac{z_i+1}{z_i-1}$, $z_\pm = \frac{\omega}{kv_{\rm F}} \pm \frac{k}{2p_{\rm F}}$, is the Lindhard function, ω, k are the energy and momentum transferred in the p–h channel.

2. SPECTRUM OF SCALAR EXCITATIONS

Solution of equation $f_0^{-1} = -\Phi(\omega,\vec{k})$ gives the spectrum of excitations in the scalar channel $\omega(k)$. For the repulsive interaction $f_0 > 0$ there exists a real solution of this equation $\omega = kv_{\rm F}s(k/p_{\rm F})$, $s(k/p_{\rm F}) \approx s_0 + s_2(k/p_{\rm F})^2, k/p_{\rm F} \ll 1$, $v_{\rm F} = p_{\rm F}/m^*$ is the Fermi velocity. This solution is called the zero sound. The zero sound mode exists as a quasi-particle mode for frequencies much larger than the fermion collision time, i.e. $\omega \gg \varepsilon_{\rm F}/T^2$, $\varepsilon_{\rm F}$ is the Fermi energy, T is the temperature. In the opposite limit the solution describes a hydrodynamic (first) sound. For $k > k_{\rm lim} \lesssim p_{\rm F}$ the spectrum branch enters the region with $\Im\Phi > 0$, and the zero sound becomes damped.

If $f_0 < 0$ the equation has purely imaginary solution which for $k \ll p_{\rm F}$ is $\omega(k) \approx i(2kv_{\rm F}/\pi) \times \left[z_f - k^2/12p_{\rm F}^2\right]$, $z_f = 1 - 1/|f_0|$. For $f_0 < -1$, $z_f > 0$ and in some interval of momenta $\Im\omega(k) > 0$. The mode becomes exponentially growing (Pomeranchuk instability). Usually this instability is treated as a spinodal instability resulting in the creation of aerosol-like mixture of droplets and bubbles. We propose a new alternative that at certain conditions the Pomeranchuk instability may lead to formation of a static Bose condensate of a scalar field.

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3. EFFECTIVE LAGRANGIAN OF SCALAR FIELD

In a fermion system with a contact interaction in scalar channel one can introduce a collective scalar bosonic field by means of the Hubbard–Stratonovich transformation [2], or by formal replacement of the contact interaction to exchange by aheavy scalar boson [3]. The effective Lagrangian of the static scalar condensate field taken in the simplest form $\varphi = \varphi_0 e^{i\vec{k}_c\vec{r}}$ can be presented as follows

$$\mathcal{L}_\varphi = -\mathrm{sgn}(f_0)\left[\left(\Gamma_0^\omega\right)^{-1} + a^2 N\Phi(0,k_c)\right] \times |\varphi_0|^2 - \frac{1}{2}\Lambda(0,k_c)|\varphi_0|^4, \quad (1)$$

where the self-interacting term is determined by the integral of four fermion Green's functions evaluated in [4] as $\Lambda(0,k_c) \approx a^4\lambda\left(1 + k_c^2/2p_\mathrm{F}^2\right)$ for $k_c \ll p_\mathrm{F}$ with $\lambda = \nu/\left(\pi^2 v_\mathrm{F}^3\right)$. Variation of the Lagrangian in φ_0 yields equation $-a^2N\tilde{\omega}^2(k_c)\varphi_0 - \Lambda(0,k_c)|\varphi_0|^2\varphi_0 = 0$, $\tilde{\omega}^2(k_c) = |f_0(k_c)|^{-1} - \Re\Phi(0,k_c)$ is an effective gap in the excitation spectrum. The amplitude of the condensate field $|\varphi_0|$ and the Bose condensate energy density term become $|\varphi_0|^2 = -\frac{N\tilde{\omega}^2(k_c)}{a^2\lambda}\left(1+\frac{k_c^2}{2p_\mathrm{F}^2}\right)^{-1}$, $E_\mathrm{b} = -\frac{N^2\tilde{\omega}^4(k_c)}{2\lambda}\left(1+\frac{k_c^2}{2p_\mathrm{F}^2}\right)^{-1}$. In the presence of the condensate the incompressibility becomes, $K = K_\mathrm{f} + K_\mathrm{b}$, $K_\mathrm{b} = n\frac{d^2E_\mathrm{b}}{dn^2}$, and, therefore, the scalar Landau parameter changes

$$f_0 \to f_0^\mathrm{tot} = f_0 + f_0^\mathrm{b} = f_0 + 3m^*K_\mathrm{b}\left[f_0^\mathrm{tot}\right]/p_\mathrm{F}^2. \quad (2)$$

Here f_0 and f_0^b are functions of k_c which is to be found from the energy minimization. In case of a weak condensate (for $|f_0+1| \ll 1$) one can use $E_\mathrm{b}[f_0^\mathrm{tot}] \approx E_\mathrm{b}[f_0]$. For a developed condensate the perturbative analysis does not work and one should solve Eq. (2) self-consistently.

4. STABILITY OF THE SYSTEM WITH CONDENSATE

Let us now study excitations on top of the condensate. Their spectrum is described by the equation $a^2N[-|f_0^\mathrm{tot}(k)|^{-1} + \Phi(\omega,k)] - \delta\Sigma_\varphi = 0$, where $\delta\Sigma_\varphi = 2\Lambda(\omega,k)|\varphi_0|^2$ includes the interaction of excitations with the condensate. Making use expression for $|\varphi_0|$, we can cast the spectrum in the form $\omega \approx i\frac{2}{\pi}\tilde{\omega}^2(k_c)kv_\mathrm{F}$ for $-1 \ll \tilde{\omega}^2(k_c) < 0$. We see that the excitations are damped: *in the presence of the condensate the Fermi liquid is free from the Pomeranchuk instability of the zero-sound-like modes*.

The p–h interaction also changes in the presence of the condensate. There appears a new term in equation for the p–h amplitude, which can be included in the renormalized local interaction as $1/f_{\mathrm{ren},0} = 1/f_0^\mathrm{tot}(k_c) + 2\tilde{\omega}^2(k_c)$. For homogenous condensate with $k_c = 0$ we have $\tilde{\omega}^2(0) = -z_f = -1 - 1/f_0^\mathrm{tot}$ and consequently $f_{\mathrm{ren},0} = -f_0^\mathrm{tot}/(2f_0^\mathrm{tot}+1)$. Thus if originally $f_0 < -1$ and therefore $f_0^\mathrm{tot} < -1$, the renormalized interaction $-1 < f_{\mathrm{ren},0} < -1/2$. Hence, *in the Fermi liquid with the condensate the first sound modes are stable*. Knowing the value $f_{\mathrm{ren},0}$ one can reconstruct the energy density $E_\mathrm{tot}(n)$ of the Fermi liquid from the differential equation

$$\mathrm{d}^2E_\mathrm{tot}(n)/\mathrm{d}n^2 = 2\epsilon_\mathrm{F}(1 + f_{\mathrm{ren},0})/(3n), \quad (3)$$

which solution should continuously match the original energy-density E_f at the values of the density where $f_0 = -1$. Note that this energy density includes both mean-field and quadratic-fluctuation contributions.

Several approximations are done in our study. In $\mathcal{L}_\varphi$ we kept only terms up to φ^4. We disregarded self-interaction of excitations on top of the condensate and neglected feedback of fluctuations on the mean field. Thus, our results are valid if on the one hand φ_0 is rather small and on the other hand fluctuations on the top of the condensate yield a yet smaller contribution.

Let us demonstrate how the scheme works on example of the isospin-symmetric nuclear matter. We consider a system of 125 nucleons which energy per particle, $\mathcal{E}_N = E_N/n - m_N$, contains the volume and surface parts. The volume part satisfies standard properties of the nuclear saturation: the density $n_0 = 0.16$ fm^{-3}, the energy per particle $\mathcal{E}_0 = -16$ MeV and the nuclear incompressibility $9K_\mathrm{f} = 285$ MeV. The inclusion of the surface term shifts the saturation density to $0.9n_0$ and the energy per particle to -13.9 MeV. Values $\mathcal{E}_N$ and f_0 are shown in Fig. 1 by short-dash lines. We see that $f_0 < -1$ in some density interval. With this f_0 we solve Eq. (2) and obtain f_0^tot and $\mathcal{E}_\mathrm{tot}^{(\mathrm{MF})} = \mathcal{E}_N + E_\mathrm{b}[f_0^\mathrm{tot}]/n$ shown by solid lines. Finally long-dashed lines show the renormalized Landau parameter $f_{\mathrm{ren},0}$ and the corresponding energy per particle $\mathcal{E}_\mathrm{tot}(n) = E_\mathrm{tot}(n)/n$ with E_tot determined by

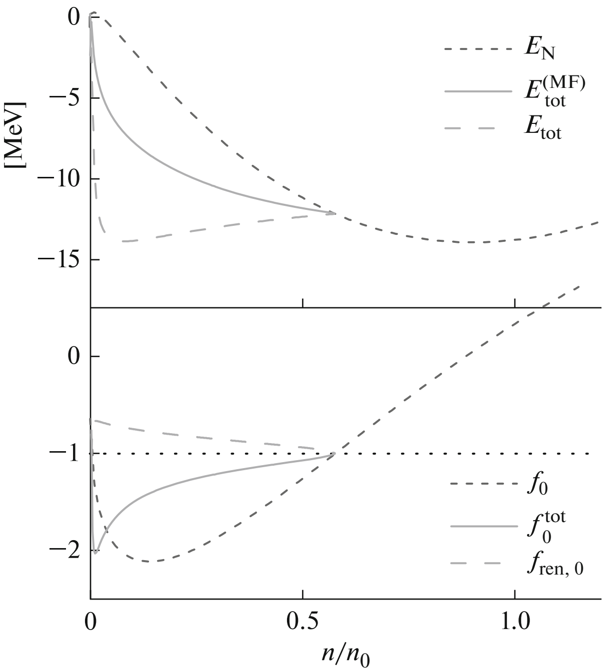

Fig. 1. Energy per particle (upper panel) and the scalar Landau parameter (lower panel) as functions of the nucleon density in a piece of isospin-symmetric nuclear matter with and without the condensate. See text for details.

Eq. (3). We see that because of the condensate formation a new metastable state can appear (in our example at density $0.081n_0$ with the binding energy 13.8 MeV).

5. CONCLUSIONS

Spectra of scalar excitations in Fermi liquid are found. Local 4-fermion interaction is bosonized and the effective Lagrangian for the scalar excitations is constructed. It is shown that the Pomeranchuck instability leads to a condensation of scalar quanta. In the presence of the condensate the instabilities in first and zeroth sounds are removed. Owing to the condensate a novel metastable state in isospin-symmetric nuclear matter might exist at nuclear subsaturation density.

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